

Application of Extended and Linear Kalman Filters for an Integrated Navigation System

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Abstract— Inertial navigation, integrated with other navigation aids such as Global Positioning System (GPS), has gained significance in recent years. The integrated navigation system can be used to determine attitude, velocity and position of the moving objects. This paper presents an approach to designing an INS/GPS based navigation system using a combination of an Extended Kalman Filter (EKF) and a Linear Kalman Filter (LKF). These include a first-stage eight-state EKF for the GPS data and a second-stage eight-state LKF for the INS/GPS integration. The advantage of this design is that we can utilize the simplicity of the LKF but keep the high performance of the EKF. Comparison and analysis of the new scheme is carried out, and it is demonstrated that the proposed configuration is useful technique.

Keywords- INS/GPS integration, Extended Kalman filter, navigation.

I. INTRODUCTION

The Global Positioning System (GPS) can provide continuous and accurate positioning with lines of sight (LOS) with more than four satellites. However, the accuracy and availability of GPS-based vehicular navigation systems are subject to the open-sky condition and degrade in the presence of signal blockage or reflected signals. An Inertial Navigation System (INS) can fill the GPS gaps to provide continuous navigation parameters (position, velocity, and attitude). An Inertial Measurement Unit (IMU) consists of a set of inertial sensors (gyroscopes and accelerometers). An INS usually refers to a combination of an IMU and a box PC that can provide navigation parameters in the navigation frame directly in real-time [1]. MicroElectroMechanical System (MEMS) based inertial sensors are preferred as the complementary component to GPS for general navigation applications [2]. However, the positioning accuracy of these low-cost inertial sensors degrades rapidly with time when GPS signals are interrupted [3].

The GPS and INS can be integrated by using Kalman filters [4]. The Kalman filter is one kind of optimal filter that can efficiently estimate the state of a process. In this paper we develop a scheme that utilized both the high performance of EKF and the simplicity of the LKF. These include a EKF for GPS data filtering and a LKF for INS/GPS integration. Comparison and analysis of the proposed scheme with others is carried out, and it is demonstrated that it is a useful technique for practical applications.

In this paper, we have proposed to use an extended Kalman filter (EKF) to improve the quality of an existing system [7]. The EKF is used to process the raw information from the GPS (without direct information such as velocity or position), which provide more accurate information of navigation parameters before feeding to the INS/GPS block. The simulated results showed that the output quality of the GPS-EKF is better than regular GPS. Consequently, the quality of whole of INS/GPS integrated system will be improved significantly. Position accuracy is improved by about 1-2 m.

The paper is organized as follows. In Section 2, we introduce the brief background of INS, GPS, and the proposed INS/GPS integration scheme. Simulation results to demonstrate the efficiency of our method are presented in Section 3. Section 4 concludes the paper with discussions on the results and remarks for future work.

II. WORKING PRINCIPLES

A. Inertial Navigation System

An INS consists of an Inertial Measurement Unit (IMU) and a navigation algorithm [5]. An IMU often measures moving object accelerations in three dimensions (i.e. a_x, a_y, a_z) and its rotations (i.e. p, q, r) in the body frame with a high update rate. Using a specific navigation algorithm (e.g. Strapdown Inertial Navigation System – SINS), these raw measurements can be processed to compute three kinds of navigation parameter: positions (X, Y, Z), velocities (U, V, W) and attitudes (ϕ, θ, ψ). Note that these parameters are computed in the navigation frame. It means that in the navigation algorithm we have to implement frame transformation. A serious problem with the navigation system based on INS is that it drifts with time. Drifts in gyroscope lead to wrong estimation of transformation matrix which in turn misrepresents navigation parameters. Not only drift error, there are several dominant error sources of INS: alignment errors, accelerometer bias, cross-talk errors of accelerometers and gyroscopes, and random noise. Thus, every INS needs a reliable and additional information system to reduce these errors. One kind of popular systems which is also mentioned in this paper is GPS.

B. Global Position System

The main task of Global Positioning System (GPS) is to provide highly accurate position and velocity of the object equipped GPS receiver [6], based on one-way ranging

technique. GPS user receives signals from at least four satellites and uses its self-generated replica to measure relative phase. Consequently, the position (latitude, longitude), velocity, altitude and a correction to the user's clock of the object can be computed using triangulation method. In the ideal case (i.e. without errors and Selective Availability), we can achieve a precision in calculating the position. However, we can not ignore the GPS ranging errors such as ionospheric, tropospheric, satellite clock errors, etc.

The pseudorange ρ_i can be expressed with the measurement equation for each satellite:

$$\rho_i = \|r - r^i\| + b + v_i \quad (1)$$

where r and r^i are the positions of the GPS receiver and the i^{th} satellite, $\|r - r^i\|$ is the ideal distance between the GPS receiver and the i^{th} satellite, b is the error caused by the clock bias at the GPS receiver, and v_i is the measurement noise that can be assumed white noise.

Solutions of the measurement equations can also be solved through iterative least square (ILS) [7] technique or Kalman filtering. The ILS offer a fast and simple calculation of the position by using each independent measurement set. In this paper, we propose to use the EKF to obtain high accuracy in position and velocity estimations.

C. Configuration of the proposed integrated navigation system

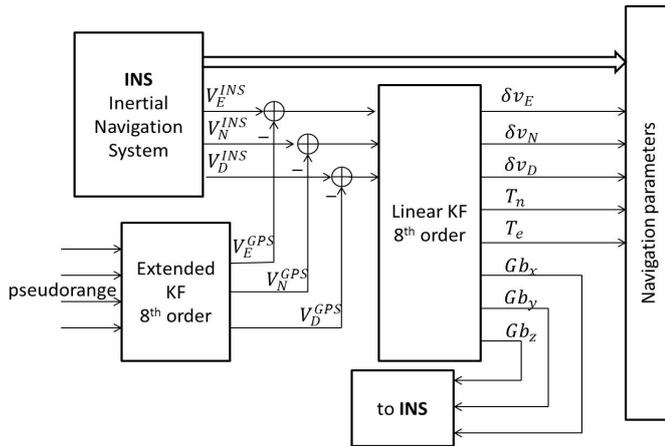


Figure 1. Configuration of the EKF and LKF in the proposed scheme

The INS system has two main advantages when comparing with other navigation system: self-contained ability and high accuracy for short-term navigation. In long-term navigation applications, the INS should work with the aid of the GPS system. The integration of INS and GPS is considered an optimal combination. The heart of integrated system is Kalman algorithm. This paper presents an approach to designing an INS/GPS based navigation system using a combination of an Extended Kalman Filter (EKF) and a Linear Kalman Filter (LKF) [8] as shown in Figure 1.

1) EKF for GPS filtering

From the pseudorange equation, it can be seen that we need to estimate the position and velocity of the GPS receiver from the measured pseudorange from at least four satellites. The state vector at time step k includes object's position (r_x, r_y, r_z), object's velocity (v_x, v_y, v_z), bias's clock b and drift's clock d :

$$x_k = [r_x, v_x, r_y, v_y, r_z, v_z, b, d] \quad (2)$$

The system model can be shown as:

$$\begin{aligned} x_k &= f(x_{k-1}, \omega_k) \\ y_k &= h(x_k, v_k) \end{aligned} \quad (3)$$

where the state transition function f is

$$f = \begin{bmatrix} r_x + \Delta t \times v_x \\ v_x \\ r_y + \Delta t \times v_y \\ v_y \\ r_z + \Delta t \times v_z \\ v_z \\ b + \Delta t \times d \\ d \end{bmatrix} \quad (4)$$

where Δt is the update rate of the GPS data (in this work, Δt can be chosen at 1 second); and ω_k is the process noise whose the covariance matrix is a 8x8 matrix:

$$Q = \begin{bmatrix} Q_x & & & \\ & Q_y & & \\ & & Q_z & \\ & & & Q_b \end{bmatrix} \quad (5)$$

where Q_x, Q_y, Q_z are the covariance matrixes of the positions and velocities in three dimentions.

$$Q_x = Q_y = Q_z = \sigma^2 \times \begin{bmatrix} \frac{\Delta t^3}{3} & \frac{\Delta t^2}{2} \\ \frac{\Delta t^2}{2} & \Delta t \end{bmatrix} \quad (6)$$

And Q_b is the covariance matrix of the clock's bias and the drift:

$$Q_b = \begin{bmatrix} S_f \times \Delta t + S_g \times \frac{\Delta t^3}{3} & S_g \times \frac{\Delta t^2}{2} \\ S_g \times \frac{\Delta t^2}{2} & S_g \times \Delta t \end{bmatrix} \quad (7)$$

where S_g, S_f are state transition variances of bias's clock and drift's clock.

In the measurement equation of (3), h is the function for the measurement (see (1)):

$$h = \sqrt{(r_x - r_x^i)^2 + (r_y - r_y^i)^2 + (r_z - r_z^i)^2} + b \quad (8)$$

Where r_k is the measurement vector at time step k which consists of pseudoranges from N satellites, and the additive observation noise in (3) has the covariance matrix R .

The EKF flowchart can be summarized in Algorithm 1.

Algorithm 1. Extended Kalman filter for GPS positioning

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Choose initial values of  $x_0, Q, R, P_0, \Delta t$ , and the total step  $N$ 
while  $k < N$ 
  1. One step projection  $\hat{x}_{k/k-1} = f(x_{k-1}, 0)$ 
  2. Linearize input function  $f$  and  $g$  to get  $F_k$  and  $H_k$  for an ordinary Kalman filter
  3. Calculate the covariance matrix of  $\hat{x}_{k/k-1}$ 
      $P_{k/k-1} = F_k P_{k-1/k-1} F_k^T + H_k Q H_k^T$ 
  4. Calculate Kalman gain
      $K_k = P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + R)^{-1}$ 
  5. Update the out state
      $\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k (y_k - H_k \hat{x}_{k/k-1})$ 
  6. Update the covariance matrix
      $P_{k/k} = (I - K_k H_k) P_{k/k-1}$ 
  7.  $k = k + 1$ 
End

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2) LKF for INS/GPS integration

The performance of LKF for INS/GPS integration has been presented in [8]. It can be shown here for the completion. We can see in Fig. 1 that the measurement vector of LKF is the subtraction of the INS velocity from GPS ones. Thus, it can eliminate the nonlinearity effect of both INS and GPS. Note that with the support of the EKF in the previous block, the variance of the measurement vector here can be significantly reduced. Thus, the performance of the overall system can be improved.

The state vector of this LKF can be shown as:

$$x_k = [T_N, T_E, \delta v_E, \delta v_N, \delta v_D, Gb_x, Gb_y, Gb_z]^T \quad (9)$$

Where T_N, T_E are the attitude errors, $\delta v_E, \delta v_N, \delta v_D$, are the subtraction of the INS velocity from GPS ones, and Gb_x, Gb_y, Gb_z , are the drift errors of three gyroscopes. Note that three first components of the state vector form the measurement vector:

$$y_k = \begin{bmatrix} \delta v_E \\ \delta v_N \\ \delta v_D \end{bmatrix} = \begin{bmatrix} v_E^{INS} - v_E^{GPS} \\ v_N^{INS} - v_N^{GPS} \\ v_D^{INS} - v_D^{GPS} \end{bmatrix} \quad (10)$$

The drift errors of three gyroscopes would be brought back to the INS in order to adjust the angular increments:

$$\begin{bmatrix} \Delta W_x \\ \Delta W_y \\ \Delta W_z \end{bmatrix} = \begin{bmatrix} \Delta W_x \\ \Delta W_y \\ \Delta W_z \end{bmatrix} - \Delta t \times \begin{bmatrix} Gb_x \\ Gb_y \\ Gb_z \end{bmatrix} \quad (11)$$

The quality of sensors used in the IMU also determines the system performance. We can see the role of these inertial sensors in the covariance matrix of the process noise.

III. IMPLEMENTATION RESULTS

Figure 2 is the velocities estimated by the EKF using the satellite positions and pseudoranges. With the assumed that the vehicle is standing still, its velocities are converged to 0 m/s that proved the correctness of the program.

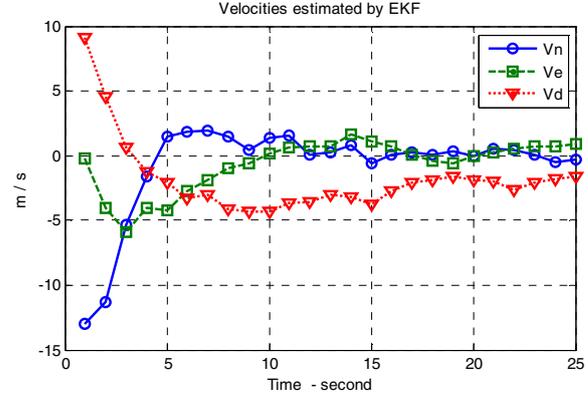


Figure 2. Velocities estimated by the EKF using the satellite positions and pseudoranges

Figure 3 shows more specific accuracy of the EKF when comparing the position error of the ILS and the EKF.

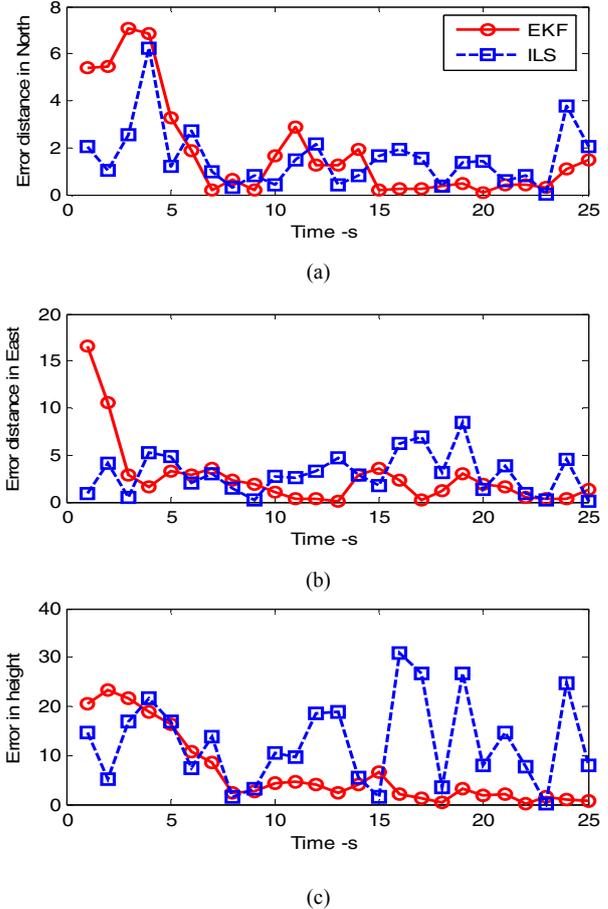
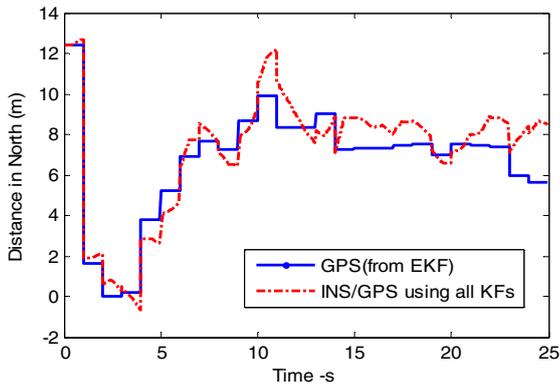
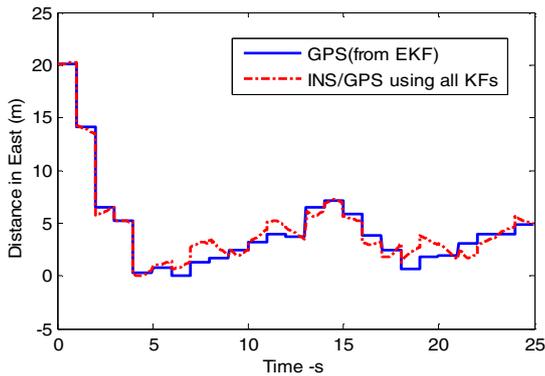


Figure 3. Position comparisons between the EKF and the ILS



(a)



(b)

Figure 4. Calculation of distances in the North and the East using only GPS and INS/GPS systems

Figure 4.a and 4.b are the position outputs along the north and east of the entire integrated system using the extended Kalman filter (to improve the accuracy of the GPS) and linear Kalman filter (to combine output of GPS with inertial navigation system). The integrated system will both have such a high accuracy (over the whole system without EKF), high updated speed with acceptable complexity.

IV. CONCLUSION

This paper has success in combination of an extended and a linear Kalman filters to a simple but high performance system. The quality of the entire system INS/GPS offer the position estimation in the range 1-2 m. There are a number of studies that have integrated parameters of the object navigation and the GPS in a single EKF filter. This approach increases the complexity of the system and the stability of the entire system will be able to affected when a certain input errors encountered. This integration system ensures the accuracy, flexibility and also reduces the complexity, as a basis favorable to be put to use in real time.

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