The shifting dependence dynamics between the G7 stock markets

Ahmed BenSaïda∗†, Sabri Boubaker‡ and Duc Khuong Nguyen§

†HEC Sousse, LaREMFiQ Laboratory, University of Sousse, Tunisia
‡Groupe ESC Troyes, Champagne School of Management, Troyes, France
‡International School, Vietnam National University, Hanoi, Vietnam
§IPAG Business School, Paris, France

(June 7, 2017)

The growing interdependence between financial markets has attracted special attention from academic researchers and finance practitioners for the purpose of optimal portfolio design and contagion analysis. This article develops a tractable regime-switching version of the copula functions to model the intermarkets linkages during turmoil and normal periods, while taking into account structural changes. More precisely, Markov regime-switching C-vine and D-vine decompositions of the Student’s t copula are proposed and applied to returns on diversified portfolios of stocks, represented by the G7 stock market indices. The empirical results show evidence of regime shifts in the dependence structure with high contagion risk during crisis periods. Moreover, both the C- and D-vines highly outperform the multivariate Student’s t copula, which suggests that the shock transmission path is as important as the dependence itself, and is better detected with a vine copula decomposition.

Keywords: Financial co-movement; Regime-switching; Vine copula; Multivariate Student’s t

JEL Classification: G15, C34, C58

1. Introduction

Stock market dependence plays an important role in the determination of optimal portfolio design and contagion analysis. A high degree of dependence between two markets reduces the benefits from diversifying internationally and implies high probability of contagion risk when a crisis shock occurs in one of these markets. Prior studies have mainly measured cross-market dependence through linear correlation, GARCH-based dynamic correlations (e.g., Asai and McAleer 2009, Colacito et al. 2011, Javed and Virk 2017), copula-based dependence coefficient (e.g., Caillault and Guégan 2005, Huang et al. 2016), and realized correlations (e.g., Ferland and Lalancette 2006, Aslanidis and Christiansen 2014). Despite its simplicity, the linear correlation approach is no longer viewed as a reliable measure for market dependence as it is static and only captures the average linkage between markets. Among the remaining models, copula functions have emerged as a promising method for dependence modeling owing to their ability to simultaneously deal with asymmetric, nonlinear, and tail dependence as documented in many recent studies (e.g., Ang and Chen 2002, Herrera and Eichler 2011, Patton 2012).

*Corresponding author. Email: ahmedbensaida@yahoo.com
This paper was written while Sabri Boubaker was visiting professor in Finance at the International School, Vietnam National University, Hanoi, Vietnam.
Since the seminal work of Sklar (1959), copula theory has rapidly grown to cover not only symmetric but also asymmetric copula families in both bivariate and multivariate settings (Jaworski et al. 2010). Copulas have become increasingly popular in finance with meaningful applications in, among others, credit risk assessments (Cousin and Laurent 2008, Crook and Moreira 2011), risk management (Kole et al. 2007, Silva Filho et al. 2014, Siburg et al. 2016), and portfolio optimization (Kakouris and Rustem 2014, Al Janabi et al. 2017). Also, they have been widely used in recent studies to model the dependence structure and contagious effects among financial markets in the aftermath of the US subprime crisis of 2007, the global financial crisis of 2007-2008, and the European debt crisis of 2009-2012 (e.g., Philippas and Siriopoulos 2013, Zhang 2014, Reboredo et al. 2016). These studies mainly document, in addition to time-varying and asymmetric co-movement, higher cross-market dependencies during turmoil periods, which suggests the superiority of optimal portfolio strategies with nonlinear dependence (Kole et al. 2007).

This article develops a Markov regime-switching C-vine and D-vine copula approach to measure the dependence structure of seven synthetic portfolios of stocks which are represented by stock market indices of G7 developed countries. It makes three main contributions to the existing literature. First, our vine copula decompositions through the estimation of the bivariate Student’s $t$ copula at each node allow for regime shifts in the dependence parameter and are, thus, flexible enough to capture a wide range of dependence, tail dependence and asymmetric dependence. As in Forbes and Rigobon (2002), we define financial contagion as “a significant increase in cross-market linkage after a shock to one country or group of countries”. Therefore, the contagion can be measured as a substantial increase in the magnitude of the copula’s dependence parameter, to the extent that the linkage can be identified as the dependence structure – whether linear or nonlinear – between markets, and the magnitude of the dependence can be modeled over different regimes – whether known or hidden – to determine the significant increase, if any. As a result, our analytical framework enables the identification of two hidden Markov regimes – a low contagion regime and a high contagion regime. Second, we use a GARCH(1,1) process with the highly flexible skewed generalized $t$ (SGT) distribution to model the marginal distributions of index returns. For instance, BenSaïda and Slim (2016) show that the SGT can nest a large variety of other distributions and provides a remarkable fit to financial returns. Finally, we compare the performance of our regime-switching vine copula model to that of the multivariate Student’s $t$ copula which has been proven to be effective in dependence modeling and risk management.\footnote{In a related study, Bauer et al. (2012) propose a combination of pair copulas and directed acyclic graph (DAG-PCC) to model both the conditional dependence and directed interactions between variables. However, the selection of a suitable non-Gaussian DAG-PCC model is challenging as it requires assumptions about the Markov structure to derive directed interactions and the identification of appropriate pair copulas.}

Our empirical investigation uses daily data of G7 stock market index prices over the period January 1, 2000 to September 30, 2016. The main results indicate that the marginal models with the SGT distribution obtain the best fits to all stock returns, which implies that the marginal model parameters can be suitably used for the estimation of copula models. We further find that the C-vine decomposition slightly outperforms the D-vine and is largely superior to the multivariate Student’s $t$ copula under the scenario of a single regime copula. When regime shifts are introduced (low versus high contagion regimes), the D-vine structure is the best copula model, followed closely by the C-vine and very far by the multivariate Student’s $t$ copula. In particular, the smoothed probabilities of the D-vine copula clearly describe the period of high contagious effects among the G7 stock markets, such as the Gulf war (2001-2002), the US subprime crisis (2007) and the subsequent global financial...
crisis (2008-2009), the European public debt crisis (end of 2009-2012), and the Brexit event on June 23, 2016.

The remainder of the paper is as follows. Section 2 describes the methodology and theoretical design of a regime-switching copula model. Section 3 reports the results and major findings. Finally, section 4 concludes.

2. Model and theoretical design

In finance literature, financial contagion between markets is generally investigated using two main approaches, namely, (1) multivariate GARCH models (Pelletier 2006), which rely on dynamic linear correlations; and (2) copula functions which, as argued by Ning (2010), capture nonlinear and asymmetric dependence. Our paper extends the second approach to the multivariate case under structural changes affecting the dynamics of dependent variables.

2.1. Copulas

A copula function measures the joint behavior between variables and can highly detect dependencies between financial markets, or more precisely, shock transmission (Horta et al. 2016).

Sklar (1959) introduced copulas, where a \( d \)-dimensional vector \( \mathbf{x} = (x_1, x_2, \ldots, x_d) \in \mathbb{R}^d \) with joint cumulative distribution function \( F \); then, there exists a copula function \( C : [0,1]^d \rightarrow [0,1] \), such that the multivariate distribution function is written in terms of univariate marginals \( F_i \), for \( i \in \{1, 2, \ldots, d\} \), and each marginal \( F_i(x_i) = u_i \) is uniformly and independently distributed on \([0,1] \):

\[
F(x_1, \ldots, x_d) = C \left( F_1(x_1), \ldots, F_d(x_d) \right)
\]

The copula \( C \) is unique when \( \mathbf{x} \) is a set of continuous random variables. From eq. (1) we can construct a valid joint distribution \( F \) for any combination of univariate distributions \( F_1, \ldots, F_d \) and any copula \( C \). The density function can be derived as follows:

\[
f(x_1, \ldots, x_d) = c_{1\ldots d}(F_1(x_1), \ldots, F_d(x_d)) \prod_{i=1}^d f_i(x_i)
\]

where \( f_i \), for \( i \in \{1, \ldots, d\} \), represent the marginal density functions, and \( c_{1\ldots d} \) is the \( d \)-dimensional copula density function defined by:

\[
c_{1\ldots d}(u_1, \ldots, u_d) = \frac{\partial^d C(u_1, \ldots, u_d)}{\partial u_1 \ldots \partial u_d}
\]

2.2. Vine decomposition

Practical implementation of multivariate copulas can be conducted through the symmetric Student’s \( t_\nu \), either jointly, or by using a vine decomposition. Bedford and Cooke (2002) have decomposed the

\[^2\text{Patton (2012) provides a detailed survey on copula models for economic time series.}\]
density of a $d$-dimensional distribution into a series of linked trees of \textit{bivariate} copulas as building blocks called \textit{vines}. A vine consists of an acyclic sequence of $d-1$ connected trees $T_j$, with nodes $N_j$ and edges $E_j$, with $1 \leq j \leq d-1$. Aas et al. (2009) have proved that vines are numerically tractable, and introduced two popular decompositions: \textit{canonical} (C), and \textit{drawable} (D) vines.

For a C-vine, the $d$-dimensional density in eq. (2) is decomposed in eq. (4), where $j$ defines the trees, $i$ the edges that link these trees, and $c_{i,j}$ is a bivariate copula density. Each tree $T_j$ has a unique node that is connected, through edges, to all other $d-j$ nodes. The C-vine is appropriate when a particular variable is known to be central in governing the interactions between other variables fig. 1.a.

$$f(x) = \prod_{k=1}^{d} f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{j,j+i,...,j-1}$$

For a D-vine, the $d$-dimensional density is decomposed in eq. (5), where each tree $T_j$ is connected to a maximum of two edges. D-vines are suitable when bivariate dependence between two variables affects the subsequent pair dependence, the structure resembles to a one-way direction path fig. 1.b.

$$f(x) = \prod_{k=1}^{d} f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j|i+1,...,i+j-1}$$

(a) C-vine decomposition.  

(b) D-vine decomposition.

Figure 1. Five-dimensional vine decomposition.
Both eqs. (4) and (5) need a fast recursive method to compute the marginal conditional distribution function $F(x_j|x)$ involved in the pair copula construction (Aas et al. 2009). Given a $d$-dimensional vector $x$, $x_j \in x$, and $x_{-j}$ denotes the $x$-vector excluding $x_j$:

$$F(x_j|x) = \frac{\partial C_{i,j}(x_{-j})}{\partial F(x_i|x_{-j})} \left[ F(x_j|x_{-j}), F(x_i|x_{-j}) \right]$$

Therefore, given the set of parameters $\theta_{u,v}$ for the joint copula distribution $C_{u,v}$, the bivariate case simplifies to the $h$-function in eq. (6), which helps recursively compute the likelihood function corresponding to a copula vine.

$$h(u|v; \theta_{u,v}) = \frac{\partial C_{u,v}(u,v; \theta_{u,v})}{\partial v} \quad (6)$$

Joe (2015) argues that vines have many desirable properties, such as the inclusion of independence and co-monotonicity, flexible and wide range dependence, flexible tail dependence, flexible tail asymmetries, closed-form density, and ease of simulation. The major assumption to construct vine copulas is that at some higher nodes, a conditional copula does not depend on the values of the variables which they are conditioned on. Hobæk Haff et al. (2010), Czado et al. (2012) assert that this simplifying assumption is “a rather good approximation”. However, Acar et al. (2012) show that this assumption can be misleading, which strongly refrains the use of vine decompositions. Stöber et al. (2013) fill up the gap between these contradicting results and show that only the Student’s $t_\nu$ and the Archimedean copulas based on the gamma Laplace transform and their extensions can be decomposed using a pair copula construction (PCC), in which the building blocks are independent of the values that are conditioned on. Therefore, our methodology consists in estimating the C-vine and D-vine decompositions with the bivariate Student’s $t_\nu$ copula at each node, and compare the results with the joint multivariate Student’s $t_\nu$ copula. This approach is of practical interest to investigate the performance of the same copula function when decomposed using vines, or when used in its multivariate form.

2.3. Multivariate Student’s $t_\nu$ copula

The symmetric $d$-variate $t_\nu$ density with correlation matrix $R$ and for $x \in \mathbb{R}^d$ is defined in eq. (7), where $\nu$ is a scalar defining the degrees-of-freedom.

$$t_{d,\nu}(x, R) = |R|^{-\frac{d}{2}} \frac{\Gamma\left(\frac{\nu+d}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \left(\pi \nu\right)^{\frac{d}{2}}} \left(1 + \frac{x'R^{-1}x}{\nu}\right)^{-\frac{\nu+d}{2}} \quad (7)$$

Hence, the multivariate $t_\nu$ copula cumulative distribution function (CDF) for $u \in [0,1]^d$ is

$$C(u, R, \nu) = T_{d,\nu}\left(T_{1,\nu}^{-1}(u_1), \ldots, T_{1,\nu}^{-1}(u_d); R\right) \quad (8)$$

where $T_{d,\nu}(\cdot)$ is the multivariate CDF of the Student’s $t_\nu$, and $T_{1,\nu}^{-1}(\cdot)$ is the univariate inverse cumulative distribution function – or quantile function. The multivariate $t_\nu$ copula density is expressed in eq. (9).
\[ c(u, R, \nu) = \frac{t_d, \nu \left( T^{-1}_{1, \nu}(u_1), \ldots, T^{-1}_{1, \nu}(u_d); R \right)}{\prod_{j=1}^{d} t_{1, \nu} \left( T^{-1}_{1, \nu}(u_j) \right)} \] (9)

In case of vine decomposition, Aas et al. (2009), Czado et al. (2012) show that marginal bivariate conditional distributions are needed for the pair copula construction. For the Student’s copula, the \( h \)-function is expressed in eq. (10), where \( \rho \) is the correlation coefficient between \( u \) and \( v \).

\[ h(u \mid v, \rho, \nu) = T_{1, \nu + 1} \left( T^{-1}_{1, \nu}(u) - \rho T^{-1}_{1, \nu}(v) \right) \sqrt{\frac{\nu + T^{-2}_{1, \nu}(v)}{\nu + 1}} \] (10)

Kendall’s \( \tau \) of the \( t_\nu \) copula is \( 2/\pi \), and the tail dependence parameter is \( \lambda = \lambda_U = \lambda_L = 2 T_{1, \nu + 1} \left( - \sqrt{\nu + 1} \frac{1 - \rho^2}{1 + \rho^2} \right) \) for both upper and lower tails, which is decreasing in \( \nu \) for fixed \( \rho \in (-1, 1) \). Therefore, the contagion is higher for lower degrees-of-freedom \( \nu \), and vice-versa.

### 2.4. Marginal distribution

The choice of an adequate conditional distribution function \( F \) in eq. (1) is crucial for the robustness of the copula model (Joe 2005, Ning 2010, So and Yeung 2014). Therefore, we select a GARCH(1,1) model under the highly flexible skewed generalized \( t \) (SGT) distribution, since BenSaïda and Slim (2016) show that the SGT can nest a large variety of other distributions, and is a remarkable fit to financial returns. Furthermore, BenSaïda (2015) develops a closed-form distribution function \( F \) of the SGT needed to conduct the copula estimation. Given stock returns \( r_{i,t}, i = 1, \ldots, d \) and \( t = 1, \ldots, T \), the marginals in eqs. (4) and (5) consist of estimating the following model:

\[
\begin{align*}
\left\{ r_{i,t} &= \varepsilon_{i,t} \sqrt{h_{i,t}} \\
   h_{i,t} &= \kappa_i + \alpha_{1,i} r_{i,t-1}^2 + \beta_{1,i} h_{i,t-1}
\right.\]
(11)

where \( \varepsilon_t \) are independently and identically distributed (i.i.d.) with mean 0 and variance 1, with standardized SGT density function:

\[ f(\varepsilon; \eta, \psi, \lambda) = \frac{\eta}{2 \theta B \left( \frac{1}{\eta}, \frac{\psi}{\eta} \right)} \left[ 1 + \frac{|\delta - \mu|^{\eta}}{(1 + \text{sgn}(\delta - \mu)\lambda)^{\eta \psi}} \right]^{-\frac{\eta + 1}{\eta}} \] (12)

with

\[
\theta = \frac{B \left( \frac{1}{\eta}, \frac{\psi}{\eta} \right)}{\sqrt{1 + 3\lambda^2} B \left( \frac{1}{\eta}, \frac{\psi}{\eta} \right) B \left( \frac{3}{\eta}, \frac{\psi - 2}{\eta} \right) - 4\lambda^2 B \left( \frac{2}{\eta}, \frac{\psi - 1}{\eta} \right)^2} \]

\[
\mu = -2\theta \lambda \frac{B \left( \frac{2}{\eta}, \frac{\psi - 1}{\eta} \right)}{B \left( \frac{1}{\eta}, \frac{\psi}{\eta} \right)}
\]
where \( \text{sgn} (\cdot) \) is the sign function, and \( B(\cdot, \cdot) \) is the beta function. The shape parameters \( \eta > 2 \) and \( \psi > 2 \), and the skewness parameter \( |\lambda| < 1 \).

From BenSaïda (2015, Theorem 1), the distribution function of the SGT has the following closed-form:

\[
F(\varepsilon_t) = \frac{1 - \lambda}{2} + \frac{\lambda + \text{sgn}(\varepsilon_t - \mu)}{2} I_{w(\varepsilon_t)} \left( \frac{1}{\eta}, \frac{\psi}{\eta} \right)
\]

with

\[
w(\varepsilon_t) = \frac{|\varepsilon_t - \mu|^{\eta}}{|\varepsilon_t - \mu|^{\eta} + [1 + \text{sgn}(\varepsilon_t - \mu) \lambda]^{\eta} \theta^{\eta}}
\]

where \( I_z(a, b) \) is the regularized incomplete beta function that satisfies \( I_z(a, b) = \frac{B_z(a, b)}{B(a, b)} \), with \( B_z(a, b) = \int_0^z t^{a-1} (1 - t)^{b-1} \, dt \) is the incomplete beta function.

### 2.5. Markov regime-switching

Let \( \{s_t\} \) be a state variable representing a Markov chain, i.e., it stands for the different hidden regimes of the time dependent variable. The variable \( \{s_t\} \) follows a first-order Markov chain with transition probability matrix \( P \), such that each element \( p_{ij} = \Pr (s_t = j | s_{t-1} = i) \) represents the probability of being in state \( j \) at time \( t \) knowing that at time \( t-1 \) the state was \( i \). In case of two regimes, low contagion (non-crisis period) and high contagion (crisis period), the state variable takes two values \( s_t = \{1, 2\} \) with:

\[
P = \begin{pmatrix} p_{1,1} & p_{1,2} \\ p_{2,1} & p_{2,2} \end{pmatrix} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}
\]

Hamilton (1990) proposes the maximum likelihood (ML) as a preferable estimation method for Markov regime-switching models. We introduce the regime-switching only in the dependence structure. Hence, the log-likelihood to be maximized is defined in eq. (15), divided into the marginals in eq. (15a) and the dependence structure in eq. (15b).

\[
L(x; \varphi; \nu) = \sum_{t=1}^T \sum_{i=1}^d \ln f_i (r_{i,t}; \varphi_d) + \sum_{t=1}^T \ln [p_{1,t} c_{k=1} (F_1 (r_{1,t,s_t}), \ldots, F_d (r_{d,t,s_t}); \nu_{k=1}| s_t = 1) + p_{2,t} c_{k=2} (F_1 (r_{1,t,s_t}), \ldots, F_d (r_{d,t,s_t}); \nu_{k=2}| s_t = 2)]
\]

where \( c_k \) is the copula density function in the \( k^{th} \) regime \( (k = 1, 2) \), and \( \nu_k \) is the \( t_{\nu} \) copula degrees-of-freedom. \( p_{k,t} = \Pr (s_t = k | I_{t-1}) \), with \( k = \{1, 2\} \), is the ex-ante probability of being in regime \( k \) at time \( t \) given the information available at time \( t-1 \). We use Hamilton (1989)’s filter to recursively compute \( p_{k,t} \) in eq. (16).

\[
p_{1,t} = \frac{p[l_c (x_{t-1} | s_t = 1) p_{1,t-1}] + (1-q) [l_c (x_{t-1} | s_t = 2) (1-p_{1,t-1})]}{l_c (x_{t-1} | s_t = 1) p_{1,t-1} + l_c (x_{t-1} | s_t = 2) (1-p_{1,t-1})}
\]

(16)
where $l_c(.)$ stands for the likelihood function of the copula dependence structure (not in logarithm form).

Eq. (15) is maximized in two-steps as recommended by Joe (2005). In the first step, we separately maximize the log-likelihoods of the marginals in eq. (15a). Next, we construct the uniformly residuals $u_{i,t} = \Phi\left(\frac{r_{i,t}}{\sqrt{h_{i,t}}}\right)$ and we maximize the copula log-likelihood in eq. (15b).

To overcome the path dependency problem and fuzzy dynamics between regimes when maximizing eq. (15b), we employ a method proposed by BenSaïda (2015), where we independently compute the copula function $c_k$ under each regime. We, then, compute the likelihood function as a weighted average between the different likelihoods under each regime multiplied by the filtered probabilities. In this case, the maximum log-likelihood function in eq. (15b) becomes tractable.

3. Empirical investigation

3.1. Data and summary statistics

The data are composed of the G7 daily major indices. These are S&P 500 (US), TSX 60 (Canada), Nikkei 225 (Japan), FTSE 100 (UK), DAX 30 (Germany), CAC 40 (France), and MIB 30 (Italy), starting from January 1, 2000 until September 30, 2016. The indices are plotted in fig. 2, and the descriptive statistics of the returns (logarithmic difference) are presented in table 1.

Table 1. Descriptive statistics of the index returns.

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
<th>No. of obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>8.9e-5</td>
<td>0.0123</td>
<td>-0.0947</td>
<td>0.1096</td>
<td>-0.1948</td>
<td>11.470</td>
<td>13055*</td>
<td>4370</td>
</tr>
<tr>
<td>TSX 60</td>
<td>0.0001</td>
<td>0.0145</td>
<td>-0.1445</td>
<td>0.1038</td>
<td>-0.7245</td>
<td>12.681</td>
<td>17402*</td>
<td>4370</td>
</tr>
<tr>
<td>Nikkei 225</td>
<td>-3.0e-5</td>
<td>0.0151</td>
<td>-0.1119</td>
<td>0.1164</td>
<td>-0.2486</td>
<td>7.3428</td>
<td>3469*</td>
<td>4370</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>-5.0e-5</td>
<td>0.0141</td>
<td>-0.1151</td>
<td>0.1222</td>
<td>-0.2260</td>
<td>11.782</td>
<td>14045*</td>
<td>4370</td>
</tr>
<tr>
<td>DAX 30</td>
<td>0.0001</td>
<td>0.0166</td>
<td>-0.0960</td>
<td>0.1237</td>
<td>-0.0902</td>
<td>7.5634</td>
<td>3787*</td>
<td>4370</td>
</tr>
<tr>
<td>CAC 40</td>
<td>-4.1e-5</td>
<td>0.0164</td>
<td>-0.1174</td>
<td>0.1214</td>
<td>-0.0517</td>
<td>8.8924</td>
<td>6306*</td>
<td>4370</td>
</tr>
<tr>
<td>MIB 30</td>
<td>-0.0002</td>
<td>0.0173</td>
<td>-0.1542</td>
<td>0.1238</td>
<td>-0.2419</td>
<td>9.0478</td>
<td>6684*</td>
<td>4370</td>
</tr>
</tbody>
</table>

† Mean is statistically not different from zero at the 5% significance level.
* Normality is rejected at the 5% significance level.

All means are statistically not significant, which explains the omission of a constant in the mean equation in eq. (11). Furthermore, individual skewness and kurtosis coefficients indicate different shapes of each return time series; and the Jarque-Bera test refutes normality for all returns, which confirms the necessity to fit the data using a highly flexible distribution to take into account the stylized facts usually observed in financial returns.

3.2. Marginal estimation

The results of the marginals are presented in table 2. All coefficients of the variance equation eq. (11) and SGT parameters are highly significant, which fortifies the validity of the selected marginal model. Furthermore, the heterogeneity in the estimated shape and skewness parameters of the SGT distribution indicates the benefits of employing highly flexible distributions to approximate the true shape of financial returns, including outliers. Indeed, as argued by BenSaïda and Slim (2016), low flexible distributions are not suitable to financial data because the whole shape of the governing distribution, and not just the tails, changes dramatically. Moreover, the transformed residuals generated from low flexible distributions through the cumulative distribution might not pass the test of uniformity.
To avoid copula model misspecification, Patton (2006) suggests that the probability integral transform (PIT) $u_{i,t}$ must be uniformly and independently distributed on the interval $(0, 1)$. Consequently, we perform the BDS test of Broock et al. (1996) and the Cramer-von Mises (CvM) test to, respectively, verify if the transformed residuals $u_{i,t}$ are i.i.d. and Unif $(0, 1)$.

The BDS test rejects i.i.d.-ness for the S&P 500, TSX 60, and DAX 30. Moreover, the CvM test rejects uniformity only for the S&P 500. Indeed, it is highly unlikely to correctly specify the dynamics generating financial returns (BenSaïda and Slim 2016). Jondeau and Rockinger (2006), Lee and Long (2009) suggest selecting the model using optimal metric distance, i.e., with lowest absolute value of
### Table 2. Marginal estimation results.

<table>
<thead>
<tr>
<th>Index</th>
<th>GARCH coefficients</th>
<th>SGT parameters</th>
<th>Max. log-likelihood</th>
<th>CvM</th>
<th>BDS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>κ</td>
<td>α₁</td>
<td>β₁</td>
<td>η</td>
<td>ψ</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>1.5e-6</td>
<td>0.0951</td>
<td>0.8954</td>
<td>1.2929</td>
<td>-0.0740</td>
</tr>
<tr>
<td></td>
<td>(4.486)*</td>
<td>(9.115)*</td>
<td>(83.56)*</td>
<td>(33.05)*</td>
<td>(2.0e7)*</td>
</tr>
<tr>
<td>TSX 60</td>
<td>9.5e-7</td>
<td>0.0679</td>
<td>0.9283</td>
<td>1.8026</td>
<td>-0.1492</td>
</tr>
<tr>
<td></td>
<td>(3.214)*</td>
<td>(9.944)*</td>
<td>(136.3)*</td>
<td>(29.44)*</td>
<td>(2.0e4)*</td>
</tr>
<tr>
<td>Nikkei 225</td>
<td>4.7e-6</td>
<td>0.0828</td>
<td>0.9283</td>
<td>1.7480</td>
<td>-0.0583</td>
</tr>
<tr>
<td></td>
<td>(4.556)*</td>
<td>(9.223)*</td>
<td>(80.32)*</td>
<td>(12.84)*</td>
<td>(2.0e4)*</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>2.5e-6</td>
<td>0.1075</td>
<td>0.8806</td>
<td>1.6372</td>
<td>-0.0797</td>
</tr>
<tr>
<td></td>
<td>(4.945)*</td>
<td>(10.53)*</td>
<td>(83.74)*</td>
<td>(30.95)*</td>
<td>(4.5e4)*</td>
</tr>
<tr>
<td>DAX 30</td>
<td>2.3e-6</td>
<td>0.0763</td>
<td>0.9157</td>
<td>1.5905</td>
<td>1.7632</td>
</tr>
<tr>
<td></td>
<td>(3.723)*</td>
<td>(9.561)*</td>
<td>(104.1)*</td>
<td>(31.88)*</td>
<td>(2.7e5)*</td>
</tr>
<tr>
<td>CAC 40</td>
<td>2.5e-6</td>
<td>0.0818</td>
<td>0.9094</td>
<td>1.5905</td>
<td>1.7632</td>
</tr>
<tr>
<td></td>
<td>(3.957)*</td>
<td>(9.614)*</td>
<td>(100.1)*</td>
<td>(13.77)*</td>
<td>(2.524)*</td>
</tr>
<tr>
<td>MIB 30</td>
<td>1.9e-6</td>
<td>0.0832</td>
<td>0.9119</td>
<td>1.7495</td>
<td>15.819</td>
</tr>
<tr>
<td></td>
<td>(3.729)*</td>
<td>(10.32)*</td>
<td>(114.2)*</td>
<td>(13.88)*</td>
<td>(2.337)*</td>
</tr>
</tbody>
</table>

*Statistically significant at 5% confidence level.

Note: This table reports the marginal univariate GARCH estimation results under the SGT distribution. *t*-statistics are in parentheses. *p*-values of the BDS and CvM statistics are in brackets. The null hypothesis of the BDS test is that the filtered residuals \( r_{i,t} \sqrt{h_{i,t}} \) are i.i.d., the statistic is computed at the second dimension. Alternatively, the null hypothesis of the CvM test is that the filtered residuals are uniform on the interval (0, 1). *p*-values are computed using Monte-Carlo simulations to avoid distributional related discrepancies. Small *p*-values cast doubt on the validity of the null hypothesis.

### 3.3. Single regime results

The results under single regime dependence structure of eq. (15b) are presented in table 3. Aas et al. (2009) show that vine structures can be decomposed in \( n!/2 \) different ways, and a proper form is critical to perform the estimation. Consequently, for C-vines, we employ a method proposed by Czado et al. (2012), by ordering the PIT \( u_i \) with respect to the sum of absolute pairwise Kendall’s tau \( \hat{\tau}_{i,j} = \sum_{d=1}^{d} |\hat{\tau}_{i,j}| \) in fig. 3 to find the maximum spanning tree. For D-vines, Aas et al. (2009) propose finding the shortest Hamiltonian path in terms of \( 1 - |\hat{\tau}_{i,j}| \), where \( \hat{\tau}_{i,j} \) is the estimated Kendall’s tau for two random variables \( u_i \) and \( u_j \).

Under a single regime, the C-vine decomposition slightly outperforms the D-vine, and largely beats the multivariate Student’s \( t_\nu \) copula. Model selection is based on the Akaike information criterion (AIC), and the Bayesian information criterion (BIC). In fact, as argued by Horta et al. (2016), the transmission path of financial contagion across markets is crucial, and the dependence itself lacks pertinent information when modeled globally without knowing the contagion path.

### 3.4. Regime-switching results

The results under regime-switching dependence structure of eq. (15b) are presented in table 4. According to the selection criteria AIC and BIC, the D-vine slightly beats the C-vine, yet both decompositions largely outperform the multivariate \( t_\nu \) copula. The regime-switching model is preferred to the single regime with highly significant transition probabilities \( p \) and \( q \).

The smoothed probabilities \( \Pr(s_t = k|I_T) \) of being in high contagion markets (regime \( k = 2 \)) at
time $t$ given the information available at time $T$ are plotted in fig. 4. These probabilities are computed using the backward-forward algorithm of Kim (1994) improved by Klaassen (2002). In a nutshell, the filtered probabilities in eq. (16) deliver estimates based on information up to time $t - 1$. This is of limited content since we have observation up to time $T$. Hence, the technique consists of making an inference about the unobserved states by incorporating the previously neglected sample information. Consequently, the smoothing algorithm gives the best estimate of the unobservable regime at any point within the sample.

Denote the estimated filtered probability vector $\hat{\xi}_{t|t} = \Pr(s_t = k|I_{t-1})$ as computed from eq. (17) for $K$ regimes (the filtered equation in eq. (16) is for 2 regimes).
Table 4. Regime-switching copula estimations.

<table>
<thead>
<tr>
<th></th>
<th>C-vine ( t_\nu )</th>
<th>D-vine ( t_\nu )</th>
<th>Multivariate ( t_\nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum log-likelihood</td>
<td>13403.9</td>
<td>13419.3</td>
<td>13231.8</td>
</tr>
<tr>
<td>Number of estimated coefficients</td>
<td>44</td>
<td>44</td>
<td>4</td>
</tr>
</tbody>
</table>

**Degrees-of-freedom \( \nu \)**

<table>
<thead>
<tr>
<th>Regime</th>
<th>fig. 5.a</th>
<th>fig. 6.a</th>
<th>13.437*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(13.904)</td>
</tr>
<tr>
<td>Regime 2 (high contagion)</td>
<td>fig. 5.b</td>
<td>fig. 6.b</td>
<td>2.737*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(11.378)</td>
</tr>
</tbody>
</table>

**Transition probability matrix**

\[
p = \begin{bmatrix} 0.9928 & 0.000 \\ 0.9959 & 0.000 \end{bmatrix}^{†} \quad q = \begin{bmatrix} 0.9763 & 0.000 \\ 0.9903 & 0.000 \end{bmatrix}^{†} \]

**Unconditional probability of regime 1**

\[
\pi_1 = \frac{1-q}{2-p-q} = \begin{bmatrix} 0.7670 & 0.7029 & 0.8628 \end{bmatrix}
\]

**Expected duration in days**

<table>
<thead>
<tr>
<th>Regime</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1 (low contagion)</td>
<td>138.4</td>
</tr>
<tr>
<td></td>
<td>246.2</td>
</tr>
<tr>
<td></td>
<td>77.8</td>
</tr>
<tr>
<td>Regime 2 (high contagion)</td>
<td>42.18</td>
</tr>
<tr>
<td></td>
<td>103.5</td>
</tr>
<tr>
<td></td>
<td>12.42</td>
</tr>
</tbody>
</table>

**Information criteria**

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-vine ( t_\nu )</td>
<td>-26719.8</td>
<td>-26439.0</td>
</tr>
<tr>
<td>D-vine ( t_\nu )</td>
<td>-26750.7</td>
<td>-26469.9</td>
</tr>
<tr>
<td>Multivariate ( t_\nu )</td>
<td>-26455.6</td>
<td>-26430.0</td>
</tr>
</tbody>
</table>

*Statistically significant at 5% confidence level. \( t \)-statistics of the degrees-of-freedom are in parentheses.

\( \dagger \) \( p \)-values of the transition probability matrix are in brackets.

\[
\hat{\xi}_{t|t} = \frac{\eta_t \odot \hat{P} \hat{\xi}_{t-1|t-1}}{1_{K,1} \left( \eta_t \odot \hat{P} \hat{\xi}_{t-1|t-1} \right)}
\]

where \( \odot \) is the element-by-element matrix multiplication, \( \eta_t = f \left( x_t | s_t = k, x_{t-1}, \theta_k \right) \) is a \((K \times 1)\) vector of conditional density functions given the parameter vector \( \theta_k \) relative to regime \( k \) and all available observations \( x_{t-1} \) up to \( t-1 \). \( 1_K \) is a \((K \times 1)\) column vector of ones, and \( \hat{P} \) designates the estimated transition probability matrix. The smoothed probability vector \( \hat{\xi}_{t|T} = \Pr(s_t = k | I_T) \) is computed in eq. (18).

\[
\hat{\xi}_{t|T} = \left[ \hat{P}' \left( \hat{\xi}_{t+1|T} \odot \hat{\xi}_{t+1|t} \right) \right] \odot \hat{\xi}_{t|t}
\]

where \( \odot \) denotes the element-by-element matrix division, and \( \hat{\xi}_{t+1|t} = \hat{P} \hat{\xi}_{t|t} \) is the forward step. The recursion is started backward with the final filtered probability vector \( \hat{\xi}_{T|T} \), and for \( t = T - 1, \ldots, 1 \).

The smoothed probabilities in fig. 4(a) of the C-vine, fig. 4(b) of the D-vine, and fig. 4(c) of the multivariate \( t_\nu \) show several periods of high contagion, during which the probabilities of being in regime 2 are close to one, mainly: (i) the dot-com bubble that ended the first quarter of 2000, showing a starting spike at the far left of each sub-figure; (ii) the Gulf war which erupted in mid 2001 and
ended in late 2002; (iii) the global financial crisis and Great Recession from the end of 2007 until the second quarter of 2009; followed directly by (iv) the European sovereign debt crisis from late 2009 until the end of 2012; and finally (v) the Brexit on June 23, 2016 with a high spike at the far right of each sub-figure. As we notice from fig. 4, the Greek depression *per se*, which ended early 2016, has no effect on the contagion between the G7 countries. Indeed, the Outright Monetary Transactions (OMT) program announced by the European Central Bank (ECB) on August 2, 2012, aiming to prevent any speculative attack in government bond markets of the Euro area and to fragment the spillover, *i.e.*, eliminate shocks transmission across markets (Ehrmann and Fratzscher 2017), has limited the crisis spillover to G7 members, leading to a low contagion since late 2012.3

The D-vine decomposition’s smoothed probabilities in fig. 4(b) clearly describe the periods of high contagion between the G7 countries, followed by the C-vine decomposition. Nevertheless, the smoothed probabilities of the multivariate copula are rather fuzzy showing frequent multiple spikes with no clear distinction of the periods of high contagion. Modeling the dependence *per se* does not provide enough information on the dynamics linking financial markets. In fact, the contagion transmission path is as important as the dependence itself. As a result, vine decompositions provide better understanding of financial contagion than the multivariate copula.

The first level trees of the C-vine and D-vine copulas under the two-regime dependence are presented in figs. 5 and 6, respectively.4 All degrees-of-freedom $\nu$ are statistically significant at 5% confidence level, except for the Nikkei 225 under the high contagion regime 2. The C-vine shows that the French market is at the center of transmitting financial depressions, followed directly by the German market. Nevertheless, the D-vine shows that the contagion starts from the German market, and follows a specified path from the French to the Japanese markets. Each regime is characterized by different copula coefficients $\nu$; the lower the degrees-of-freedom, the higher the contagion.

4. Conclusion

The level of cross-market dependence has direct implications for optimal portfolio design and policies aiming at preventing harmful shock transmission and contagion risk from increased financial integration over recent years. In this article, we particularly show the relevance and usefulness of Markov regime-switching C-vine and D-vine copula decompositions with flexible marginal distribution in measuring the multivariate dependence structure across international equity markets. The proposed method is naturally more promising and effective than the existing approaches including, among others, the linear correlation and GARCH-based conditional correlations, because it accurately captures the stylized facts of financial returns through the flexible skewed generalized $t$ distribution, and explicitly accounts for the presence of regime shifts in the dependence patterns between markets under consideration.

Our results indicate a clear preference for regime-switching model as well as a large superiority of the vine copula structures over the multivariate Student’s $t$ copula that was used as a benchmark model. They also suggest the suitability of highly flexible distributions to model the dynamics of stock returns. All in all, our model can be used for multi-asset portfolio optimization when the dependence structure is nonlinear and exposed to structural changes.

Some limitations could, however, be addressed in future researches. Indeed, a closer look into the results shows that vine decompositions produce different dependence dynamics and contagion mechanisms. For instance, according to C-vine, the French market is found at the center of the interactions between other markets, while in the D-vine decomposition structure, the shock transmission rather follows a one way path starting form the German market. Hence, to select the transmission path, researchers tend to favor the decomposition that yields the optimal model selection criterion. When

---

3This does not mean that the contagion across Euro area countries has been controlled, a thorough investigation is needed.

4There are 6 trees for each vine; for presentation’s purpose, we report only the first ones. The remaining trees are available from the authors upon request.
regime shifts are introduced in our study, the information criteria (AIC and BIC) jointly point to the choice of the D-vine decomposition structure.

Future research could improve our methodological approach by investigating, for example, the shock transmission path from one regime to another. This consists in redefining the contagion not only as an increase in cross-market linkage after a shock affecting one country or group of countries, but also as a change in the linkage itself, \textit{i.e.}, the transmission path differs during crisis periods than during normal periods.
Figure 5. Regime-switching C-vine (Tree 1).

Note: This figure represents the first tree of the C-vine decomposition under both regimes. The contagion transmission path begins from CAC 40, followed by DAX 30, and so on moving clockwise until Nikkei 225. Each arrow bears the estimated degrees-of-freedom of the bivariate \( t \) copula. t-statistics of the estimated coefficients are in parentheses.

* Significant at 5% confidence level.

(a) Regime 1 (low contagion).

(b) Regime 2 (high contagion).

References


(a) Regime 1 (low contagion).

(b) Regime 2 (high contagion).

Figure 6. Regime-switching D-vine (Tree 1).

Note: This figure represents the first tree of the D-vine decomposition under both regimes. The contagion transmission path begins from DAX 30 moving clockwise until Nikkei 225. Each arrow bears the estimated degrees-of-freedom of the bivariate $t_\nu$ copula. t-statistics of the estimated coefficients are in parentheses.

* Significant at 5% confidence level.

Finance 32, 63–79.


