# PHOTON AND GRAVITON: WHAT IS THE DIFFERENCE?

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## 1. INTRODUCTION

The problem of whether the gravitational waves exist in nature has not yet bee solved. At the present time, there are no any convincing proof of their experimental observaton. The gravitational waves in the linear approximation have been considered by Einstein [1] and then by a lot of researchers [2]. Apart from this, the exact solution of the Einstein equation in th form of a nonlinear plane waves were obtained by Bondi, Robinson, Trautman (see e.g. [3]). So it seems that there are no theorical objections to their existence. However, there is a problem with the definition of energy of linear and nonlinear gravitational wave, connected with the general problem of energy and time in gravitational theory [2].

In the Einstein theory with the general coordinate transformation group, the concepton of the energy can be defined in the Dirac-Arnowitt-Deser-Misner (ADM) metric [4-9]. Only for this metric one can apply the Hamiltonian description and determine the constraints. In th present paper, to determine the dynamical content of gravitational waves and their energy we xplicitly resolve these constraints and calculate the reduced action.

The application of this approach to the relativistic particle model is quite simple. The esolving of the mass-shell constraints for a relativistic particle

$$\mathcal{H} = \frac{1}{2}(-p_0^2 + p_i^2 + m^2) = 0, \tag{1.1}$$

leads to the notion of particle energy

$$p_0 = \pm\omega\ ,\quad \omega = \sqrt{p^2 + m^2}\,, \tag{1.2}$$

and resolution of the equation of motion of the corresponding coordinate gives us the denition of the observable time. We shall deal with this analogy and show that the resolution of constraints and corresponding equations of gravity leads to the notion of "spectral energy" of the typ of (1.2) and spectral time as a variable canonically conjugated to this energy.

We are convinced that the reduced action for the BRT wave contains only the kintic term and is dynamically equivalent to the Misner anisotropic excitation. In the reduced theory the BRT "graviton" represents a plane wave in metric's space and cannot be treated as a paticle like exicitation, in contrast with QED "photon"' solution which is the oscillator excitation in the field space.

The present paper is organized as follows. In section 2, we discuss the exact soluton of the Einstein solution of the type of nonlinear plane wave [3]. In section 3, using the method "gaugeless reduction" [10-14] we calculate the reduced action and spectral Hamiltonian for the ystem of graviton and electromagnetic fields. The later is used as a test of correct reduction. In section 4, we compare the dynamics of a "photon" excitation with a nonlinear graviton one.

## 2. NONLINEAR PLANE WAVE

Let us consider the Einstein equation

$$G^{\mu}_{\nu} \equiv^{(4)} R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu(4)}_{\nu} R = 0 \tag{2.1}$$

for the Bondi – Robinson – Trautman nonlinear plane wave [2, 3] propagating in the direction $x^3 = z$ with the metric

$$(ds)^2 = (dt)^2 - (dz)^2 - \chi^2 h_{AB} dx^A dx^B \quad (x^1 = x,\ x^2 = y), \tag{2.2}$$

$$h_{AB} = \begin{pmatrix} e^{2\varphi}(1+\gamma^2) & \gamma \\ \gamma & e^{-2\varphi} \end{pmatrix},\quad det\, h = 1,\ \sqrt{-g} = \chi^2, \tag{2.3}$$

where $\chi$, $\varphi$, $\gamma$ are the functions depending on $t + z = t^{(+)}$.

The Einstein equations $G^0_0 = 0$, $G^0_3 = 0$ which are the constraints have the form

$$\begin{cases} 4\dfrac{\chi''}{\chi} + \mathcal{H}_{phot} = 0 \\ 4\left(\dfrac{\dot{\chi}'}{\chi}\right)^2 + (D_t\gamma . D_z\gamma) + 4\dot{\varphi}\varphi' = 0\,, \end{cases} \tag{2.4}$$

where $\mathcal{H}_{phot}$ is the photon - like energy

$$\mathcal{H}_{phot} = \frac{1}{2}\left[(D_t\gamma)^2 + (D_z\gamma)^2\right] + 2(\dot{\varphi}^2 + \varphi'^2) \tag{2.5}$$

and the following notions for derivatives are used

$$D_t\gamma = \dot{\gamma} + 2\dot{\varphi}\gamma,\quad D_z\gamma = \gamma' + 2\varphi'\gamma, \tag{2.6}$$

$$\partial_t f = \dot{f},\quad \partial_z f = f'. \tag{2.7}$$

The equations of motion (2.1) for the metric components $\chi$, $\varphi$, $\gamma$ satisfying the constraint (2.4) are identically fulfilled due to their dependence only on the combination $t+z = t^{(+)}$ [2, 3]. We would like to note that this statement is true only for the exact theory. In the linear aproximation, ($\chi = 1$) these equations are independent, and the constraint (2.4) transforms to the condition of vanishing of the photon - like energy

$$\mathcal{H}_{phot} = 0\,.$$

So, we face the following dilemma: we have the linear gravitational wave with zero energy, or the nonlinear one with equations of motion as constraints.

The question arises: What is the energy of the nonlinear plane waves? To determine this energy we use the method of reduction of gravity based on the explicit resolution of the constraints [10, 11].

## 3. REDUCTION OF EINSTEIN'S GRAVITY

### 3.1. The Hamiltonian form

The method of gaugeless reduction [10 – 14] is useful to determine the spectrum of excitations in QED and relativistic quantum mechanics. It has been shown that after the reduction both the theories contain only the observable gauge invariant variables (two transverse plhoins and the "time–reparamtrization" invariant physical coordinates and spectral time respectivel). This method has been applyed to gravity in refs [14]. Here, we repeat this method to comare the photon and graviton excitations. We start with the conventional scalar curvature action icluding the electromagnetic field

$$W[g, A] = -\int d^4X\sqrt{-g}\left(\frac{1}{2\kappa^2}{}^{(4)}R(g) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right). \tag{3.1}$$

It is well known that the Einstein equations

$$\frac{\delta W}{\delta g_{0\mu}} = 0$$

are the Lagrange constraints. In the Hamiltonian approach they correspond to the secondary constraints, and the reduction consists in their explicit resolving with respect to the definite mmentum and coordinate.

The Hamiltonian approach with the instant form of dynamics enforces us to assume hat the space time manifold $\mathcal{M}$ can be represented as $\mathcal{M} = \mathcal{R}\times\Sigma$, where $\Sigma$ is the three dimensionasurface. The space time foliation is realized by introducing the so called embedding variables $X(, t)$ [15] which are maps from a point $x$ of the surface $\Sigma$ to a space time point $X$ of the manifol $\mathcal{M}$, and $t$ labels the leaves of the foliation. This foliation leads to the well known Dirac - Arnowit - Deser - Misner (Dirac - ADM) metric [4]

$$ds^2 = N^2(dt)^2 - a^2h_{ik}(dx^i + N^i dx^0)(dx^k + N^k dx^0), \tag{3.2}$$

where $N$ is the lapse function, $N^i$ is the shift vector, $a$ is the "scale space" componer metric, $h_{ik}$ is the "graviton component" with determinant equal to unity:

$$\sqrt{-g} = N a^3, \ \det(h_{ik}) = 1, \ a = \exp\mu. \tag{3.3}$$

The Einstein - Hilbert action (3.1) in terms of this metric possesses the manifes smmetry under the following group of transformations [5]

$$\begin{aligned} t \to t' &= t'(t), \\ x^i \to x^{i'} &= x^{i'}(t, x^1, x^2, x^3). \end{aligned} \tag{3.4}$$

Let us rewrite the action (3.1) in terms of the embeddings. The scalar curvature can be leomposed into three terms: the "kinetic" $\mathcal{K}$, the three dimensional curvature ${}^{(3)}R$, and the "surac" $\Sigma$:

$${}^{(4)}R = -\mathcal{K} + {}^{(3)}R + 2\Sigma, \tag{3.5}$$

$$\mathcal{K} = -6\frac{\overset{\odot}{\mu}{}^2}{N^2} + \frac{\overset{\odot}{h}{}^2}{4N^2}, \tag{3.6}$$

$${}^{(3)}R = \frac{4}{a^2}\left[h^{kl}\nabla_l\partial_k\mu + \frac{1}{2}\partial_k\mu\,\partial^k\mu\right] + \frac{1}{a^2}R(h), \tag{3.7}$$

$$\Sigma = \frac{1}{Na^3}\left[\partial_k(a\partial^k N) - 3\check{\partial}_0\left(\frac{a^3\overset{\odot}{\mu}}{N}\right)\right], \tag{3.8}$$

where we introduce the notation

$$\check{\partial}_{0\varphi} = \partial_0 \varphi - \partial_k(\varphi N^k) \tag{3.9}$$

$$\overset{\odot}{\mu} = \dot{\mu} - \frac{1}{3a^3}\,\partial_k(a^3 N^k) \equiv \frac{1}{3a^3}\,\check{\partial}_0(a^3)\,, \tag{3.10}$$

$$\overset{\odot}{h}{}^i_k = h^{il}\left(\dot{h}_{kl} - \nabla_l N_k - \nabla_k N_l + \frac{2}{3}\,k_{kl}\,\partial_i N^i\right), \tag{3.11}$$

$$R(h) = \frac{1}{4}\,\partial_i h^k{}_l(\partial^i h^l{}_k - 2\partial^l h^i{}_k) + \partial_k \partial_l h^{kl}\,. \tag{3.12}$$

The canonical momenta conjugated to $\mu$, $h$, and $A$ are the following:

$$P_{(\mu)} = \frac{\partial \mathcal{L}}{\partial \dot{\mu}} = -\frac{6a^3}{N\kappa^2}\,\overset{\odot}{\mu}, \tag{3.13}$$

$$P^k_{(h)^l} = \frac{\partial \mathcal{L}}{\partial \dot{h}^l_k} = \frac{a^3}{4N\kappa^2}\,\overset{\odot}{h}{}^k_l\,, \tag{3.14}$$

$$P_{(A)k} = \frac{\partial \mathcal{L}}{\partial \dot{A}^k} = \frac{a}{N}(\dot{A}_k - \partial_k A_0 - N^l F_{lk}) = \frac{a}{N}\,\overset{\odot}{A}_k\,. \tag{3.15}$$

Here $\partial_j h^k{}_l = h^{ki}\partial_j h_{il}$, $N_l = h_{li}N^i$ and $\nabla_l$ is a covariant derivative in metric $h_{ik}$.

In terms of variables:

$$\varphi = (h,\, A)\,,\quad P_{(\varphi)} = (P_{(h)},\, P_{(A)})\,,\quad \mu,\ P_{(\mu)}$$

the action (3.1) has the form:

$$W = \int d^3x\,dt\left[\left(\sum_{\varphi=(h,\,A)} P_\varphi \overset{\odot}{\varphi}\right) + P_{(\mu)}\overset{\odot}{\mu} - N\mathcal{H}_E - S_\Sigma\right].$$

Here we keep the surface term (3.8) taking into account (3.13)

$$S_\Sigma = N a^3\,\frac{\Sigma}{\kappa^2} = \check{\partial}_0 \frac{P_{(\mu)}}{2} + \frac{\partial_k(a\partial^k N)}{\kappa^2} \tag{3.17}$$

in contrast with the conventional ADM approach where this term is omitted; $\mathcal{H}_E$ is the Einstein energy density

$$\mathcal{H}_E = a^3\left[-\frac{\kappa^2}{2.6}\frac{P^2_{(\mu)}}{a^6} + T^0{}_0(h,\,A)\right], \tag{3.18}$$

where $T^0{}_0(h,\,A) \equiv T^0{}_0(h) + T^0{}_0(A)$ is the zero–zero component of the energy momentum tensor

$$T^0{}_0(h) = \frac{4\kappa^2 P^2_{(h)}}{a^6} + \frac{R(h)}{2\kappa^2 a^2}\,,\quad T^0{}_0(A) = \frac{1}{2a^4}\left(E_i E^i + \frac{F_{ij}F^{ij}}{2}\right). \tag{3.19}$$

The action (3.16) can be represented in a more detailed form as

$$W = \int d^3x dt\left[\left(\sum_{\varphi=(h,\,A)} P_{(\varphi)}\dot{\varphi}\right) + P_{(\mu)}\dot{\mu} + A_0\partial_k E^k - N\mathcal{H} + N^k \mathcal{P}_k - \frac{\dot{P}_{(\mu)}}{2} - \partial_k S^k\right]. \tag{3.20}$$

In Eq. (3.20) the surface term is

$$S^k = -\frac{a}{\kappa^2}\partial^k N + 2P^k_{(h)l}N^l - \frac{1}{6}N^k P_{(\mu)} + A_0 E^k \,, \tag{3.21}$$

and $A_0$, $N$, $N^k$ are considered as Lagrange factors for constraints

$$\mathcal{H} = 0\,, \;\; \mathcal{P}_k = 0\,, \;\; \partial_k P^k_{(A)} = 0\,, \tag{3.22}$$

where $\mathcal{P}_k$ is the Einstein momentum density:

$$\mathcal{P}_k = \frac{a^3}{3}\partial_k\left(\frac{P_{(\mu)}}{a^3}\right) + Na^3 T^0{}_k(h,\, A)\,, \tag{3.23}$$

$$T^0{}_k(h,\, A) \equiv T^0{}_k(h) + T^0{}_k(A)\,, \tag{3.24}$$

$$T^0{}_k(h) = \frac{1}{Na^3}\left[2\partial_l P^l_{(h)k} - \partial_k h^i_l P^l_{(h)i}\right]\,, \;\; T^0{}_k(A) = \frac{1}{Na^3}F_{kl}E^l\,. \tag{3.25}$$

To complete, we should like to note that the expression

$$T^k{}_k(h,\, A) \equiv T^k{}_k(h) + T^k{}_k(A)\,, \tag{3.26}$$

$$T^k{}_k(h) = -3\frac{4\kappa^2 P^2_{(h)}}{2a^6} + \frac{R(h)}{2\kappa^2 a^2}\,, \;\; T^k{}_k(A) = -T^0{}_0(A) \tag{3.27}$$

is the spatial trace of the total energy–momentum tensor for gravitons $(h)$ and photons $(A)$ in the Einstein equation $\frac{\delta W}{\delta \mu} = 0$. On the constraints (3.22) this equation has the form (see also[14])

$$S_\Sigma = Na^3\left[T^0{}_0(h,\, A) + \frac{trT(h,\, A)}{2}\right]\,, \;\; (trT = T^0{}_0 + T^k{}_k)\,. \tag{3.28}$$

The action (3.20) describes the generalized Hamiltonian dynamics for $(\mu,\, h_{kl},\, A_k)$ and $P_{(\mu)}$, $P_{(h)}$, $P_{(A)}$ with constraints (3.22).

### 3.2. Reduction of phase space

We shall act in direct analogy with the relativistic particle case and QED. As we see, in these cases the resolution of constraints leads to the construction of gauge invariant variables (QED) and to the observable time as a global invariant of the reparametrization group (relativistic particle). The same gauge invariant variables for gravity have been constructed in the framework of the cosmological perturbation theory, with the choice of the conformal time [16]. Here, we discuss the dynamical aspect of this gaugeless reduction connected with the construction of the "spectral Hamiltonian" and "spectral time", by the resolution of the "energy" constraint $\mathcal{H}_E = 0$ with respect to the space–scale momentum $P_{(\mu)}$ by analogy with the "instant form" of dynamics of relativistic particle. We should like to note that there is possibility top chose another form of dynamis which corresponds to resolution of the energy constraint with respect to different momentum. And these forms can be nonequivalent.

The explicit resolution of the constraint $\mathcal{H}_E = 0$ allows us to represent $P_{(\mu)}$ as a functional from the physical variables $\varphi$, $P_{(\varphi)}$, $\mu$. This constraint has two solutions

$$P_{(\mu)} = \mp F\,, \;\; F = \frac{\sqrt{12}}{\kappa}a^3\left[T^0{}_0(h,\, A)\right]^{\frac{1}{2}}\,. \tag{3.29}$$

The quantity in the square root in eq. (3.20) is not positive definite. In the classical case the positive values of $T^0{}_0(h, A)$ restrict the admissible regions for the physical variables $h$, $A$ and their momenta. While in quantum theory, the negative values of $T^0{}_0(h, A)$ can lead to the physical phenomena of the type of tunnel effect.

According to the reduction procedure let us sustitude the solution (3.29) to the initial action (3.16) and consider two terms $P_{(\mu)}\overset{\odot}{\mu} - S_\Sigma$. On the constraints-shell (3.22) the total derivative in $S_\Sigma$ (3.17) can be represented as

$$\breve{\partial}_0 F \equiv \frac{\partial F}{\partial \mu}\breve{\partial}_0\mu + \frac{\partial F}{\partial(\partial_i\mu)}\breve{\partial}_0\partial_i\mu + \frac{\partial F}{\partial(\Delta\mu)}\breve{\partial}_0\Delta\mu + \sum_{\varphi}\left[\frac{\partial F}{\partial\varphi}\breve{\partial}_0\varphi + \frac{\partial F}{\partial(\partial_i\varphi)}\breve{\partial}_0(\partial_i\varphi) + \frac{\partial F}{\partial P_{(\varphi)}}\breve{\partial}_0(\partial P_{(\varphi)})\right]. \tag{3.30}$$

One can see that (3.30) in the action (3.16) represents the sum of the equations of motion for the fields $\mu$, $\varphi$, $P_{(\varphi)}$:

$$S_\Sigma \equiv \underline{N}a^3\left[T^0{}_0(h,\ A) + \frac{trT(h,\ A)}{2}\right] + \frac{1}{2}\sum_{\varphi}\left[\frac{\partial F}{\partial\varphi}\breve{\partial}_0\varphi + \frac{\partial F}{\partial(\partial_i\varphi)}\breve{\partial}_0(\partial_i\varphi) + \frac{\partial F}{\partial P_{(\varphi)}}\breve{\partial}_0(\partial P_{(\varphi)})\right] \tag{3.31}$$

like the identity $\dot{\omega} \equiv \sum \dot{p}_i \frac{\partial W}{\partial p_i}$ in eq. (1.1) represents the sum of the equations of motion for a relativistic particle. $\underline{N}$ denotes the lapse function $N$ on the constraint-shell $\mathcal{H}_E = 0$:

$$\underline{N} = \overset{\odot}{\mu}\,\frac{6a^3}{\kappa^2 F} \equiv \overset{\odot}{\mu}\,\frac{\sqrt{3}}{\kappa\sqrt{T^0{}_0}}\,. \tag{3.32}$$

The identity (3.34) means that the classical dynamics of the metric (3.13), (3.28), (3.29) defines the dynamics of the "matter" field (in our case – gravitons and photons). This fact was discovered by V. Fock [17] and then rediscovered by a number of authors.

By using eq. (3.32) and eq. (3.29) in the form

$$F\overset{\odot}{\mu} = \underline{N}\,\frac{\kappa^2 F^2}{6a^3} = \underline{N}a^3\, 2T^0{}_0(h,\ A)\,, \tag{3.33}$$

we can easily get the result for the reduced action (3.16) on the constraint-shell $\mathcal{H}_E = 0$

$$W^{Red}_{\pm} = \int d^3x dt\left\{\sum_{\varphi}\left[P_{(\varphi)}\overset{\odot}{\varphi} \mp \frac{1}{2}\frac{\partial F}{\partial\varphi}\breve{\partial}_0\varphi \mp \frac{1}{2}\frac{\partial F}{\partial(\partial_i\varphi)}\breve{\partial}_0(\partial_i\varphi) \mp \frac{1}{2}\frac{\partial F}{\partial P_{(\varphi)}}\breve{\partial}_0(P_{(\varphi)})\right] \mp \underline{N}\mathcal{H}_P\right\}, \tag{3.34}$$

where

$$\mathcal{H}_P = a^3\left(T^0{}_0(h,\ A) - \frac{trT(h,\ A)}{2}\right) \equiv a^3\left[T^0{}_0(A) + \frac{4P^2_{(h)}\kappa^2}{a^6}\right]. \tag{3.35}$$

Note that lapse function (3.32) and Hamiltonian (3.35) have the analogy with the mass-shell energy and lapse function for a relativistic particle (1.1), (1.2):

$$\mathcal{H}_P = (p^2 + m^2)\,,\quad \underline{N} = \frac{\dot{x}_0}{\sqrt{p^2 + m^2}}\,. \tag{3.36}$$

We can see that the reduced action on the solution of classical equation leads to the local part of the Tolman energy momentum tensor [18] for "matter" field ($M$) including gravitons [14]

$$T_{(Tolman)}{}^{\mu}_{\nu}(h,\ M) = T^{\mu}_{\nu}(h,\ M) - \delta^{\mu}_{\nu}\,\frac{trT(h,\ M)}{2}\,. \tag{3.37}$$

Expressions (3.34), (3.35) can be considered as the basis for the construction of the Hamltonian scheme in terms of gauge invariant variables. In the flat space–time limit the reduced Hamltonian (3.35) coincides with the one for electrodynamics. It is achieved b y taking into account bth the terms $P_{(\mu)}\overset{\odot}{\mu} - S_\Sigma$, in the contrast with the conventional ADM scheme (e.g. see [7]), were the total derivative (3.17) in the initial Einstein action (3.1) is neflected and the term $\dot{\mu}P_{(\mu}$ is not considered as the part of the Hamiltonian.

## 4. DYNAMICAL CONTENT OF NONLINEAR PLANE WAVE

The nonlinear plane wave metric (2.1) and the above considered general ADM metic (3.3) are connected in the following way

$$h_{ij} = \begin{pmatrix} a h_{AB} & 0 \\ 0 & a^{-2} \end{pmatrix} , \quad \chi^2 = a^3 , \quad N^k = 0 , \quad N = 1 . \tag{4.1}$$

Let us introduce the new variable $\underline{\gamma}$

$$\underline{\gamma} = e^{-2\varphi(t^{(+)})} \int_0^{t^{(+)}} dt^{(+)'} e^{2\varphi(t^{(+)'})} \dot{\gamma}(t^{(+)'}) \tag{4.2}$$

to diagonalize the kinetic terms (3.6)

$$K = -4 \left(\frac{\dot{\chi}}{\chi}\right)^2 + 2\dot{\varphi}^2 + \frac{1}{2}\dot{\underline{\gamma}}^2 . \tag{4.3}$$

In terms of momenta $P_{(\varphi)}$, $P_{(\underline{\gamma})}$, for metric field $\varphi$, $\underline{\gamma}$ the constraint $\mathcal{H}_E = 0$ aid educed action (3.34) became

$$\mathcal{H}_E = 0 \Rightarrow 4\frac{\chi''}{\chi} + \frac{\kappa^4 [P^2_{(\varphi)} + 4P^2_{(\underline{\gamma})}]}{\chi^3} = 0 , \tag{4.4}$$

$$W^{Red}_{\pm} = V^a \int dt^{(+)} \Big[ P_{(\varphi)}\dot{\varphi} + P_{(\underline{\gamma})}\dot{\underline{\gamma}} \mp \frac{1}{2}\Big(\frac{P^2_{\varphi} + 4P^2_{\gamma}}{\chi^2}\Big)\kappa^2 \Big] . \tag{4.5}$$

The reduced equations of motion are the conservation law of momenta

$$\dot{P}_{\varphi} = 0 , \quad \dot{P}_{(\varphi)} = 0 .$$

The solution of these equations and constraint (4.4) is

$$\chi^2 = \kappa^2 \sqrt{P^2_{\varphi} + 4P^2_{\gamma}} t^{(+)} . \tag{4.6}$$

So, instead of (4.5) we get

$$W^{Red}_{\pm} = V^a \int dt^{(+)} \Big[ P_{(\varphi)}\dot{\varphi} + P_{(\underline{\gamma})}\dot{\underline{\gamma}} \mp \frac{1}{2}\sqrt{P^2_{\varphi} + 4P^2_{\gamma}} \frac{1}{t^{(+)}} \Big] . \tag{4.7}$$

Or passing to the time $\log t^+ = 2\log\chi = 3\mu$, $\mu = \ln a$ $(a^3 = \chi^2)$ we get the action of he Misner anisotropic excitations [19]

$$W^{Red}_{\pm} = V^a \int dt^{(+)} \Big[ P_{(\varphi)}\dot{\varphi} + P_{(\underline{\gamma})}\dot{\underline{\gamma}} \mp \frac{1}{2} 3\sqrt{P^2_{\varphi} + 4P^2_{\gamma}} \Big] , \tag{4.8}$$

where $\mu$ plays the role of spectral time and $\frac{3}{2}\sqrt{P_\varphi^2 + 4P_\gamma^2}$ of spectral energy, and $P_{(\varphi)}$ and $P_\gamma$ are constants. The function $\varphi$ and $\gamma$ in metric (2.1) have the form

$$\varphi = \mp\frac{3}{2}\frac{P_\varphi}{\sqrt{P_\varphi^2 + 4P_\gamma^2}}\mu, \tag{4.9}$$

$$\gamma = \mp 4\frac{P_\gamma}{P_\varphi}\left(1 - e^{-2\varphi}\right). \tag{4.10}$$

Now we can pass to quantum theory. One can define the spectral decomposition of the probability amplitude to find our quantum system in the configuration $\varphi$, $\gamma$ at the spectral time $\mu$, if at the spectral time $\mu = 0$, it was in the configuration $\varphi = \gamma = 0$

$$\psi(\varphi, \gamma, \mu) = \int dP_{(\gamma)} dP_{(\varphi)} \left(A^{(+)}_{(P)} e^{iW^{cl}_{(+)}} + A^{(-)}_{(P)} e^{iW^{cl}_{(-)}}\right),$$

where $W^{cl}_{(\pm)}$ are the actions on the classical trajectories

$$W^{cl}_{(-)} = P_{(\varphi)}\varphi + P_{(\gamma)}\gamma \mp \frac{3}{2}\sqrt{P_\varphi^2 + P_\gamma^2}\,\mu,$$

$A^{(+)}$ and $A^{(-)}$ are the creation and annihilation operators of the Misner excitation.

The considered case points out the physical equivalence of the nonlinear plane wave [3] and the Misner anisotropic excitation [19], and testifies to the fact that a graviton does not disturb the space metric, as its action contains only the kinetic term, and it looks like a plane wave in the metric space.

## DISCUSSION

To understand the difference between photon and graviton, we have considered the nonlinear gravitational wave in the Dirac-ADM metric. We got the corresponding reduced Hamiltonian which generalizes the Tolman Hamiltonian [18]. Tolman has shown [18] that this Hamiltonian leads to experimental observable difference of the behaviour of a photon and massive particle in the static gravitational field, and our generalisation of this Hamiltonian leads to the conventional description of the photon as oscillator – like excitations. The nontrivial peculiarity of this reduced Hamiltonian is the absence of the potential energy for a graviton. This means that a graviton represents the plane wave in the metric space. The question arises if how we can observe this "graviton".

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## PHOTON VÀ GRAVITON: SỰ KHÁC NHAU NHƯ THẾ NÀO

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Trên cơ sở lời giải chính xác các phương trình liên kết của trường hấp dẫn phương pháp rút gọn trường đã được dùng để phân tích bản chất động lực của sóng phẳng phi tuyến Bondi-Robinson-Trautman. Ở đây đã chứng minh rằng tác dụng rút gọn (tác dụng dựa trên lời giải của các phương trình liên kết) đối với sóng BRT chỉ chứa số hạng động năng và tương đương một cách động lực với kích thích dị hướng Misner. Như vậy, trong lý thuyết lượng tử 'graviton" BRT biểu hiện là sóng phẳng trong không gian metric và không thể luận giải bằng hạt giống như kích thích. Điều này khác với điện động lực học lượng tử lời giải "photon" là kích thích dao động tử trong không gian trường.